Monte Carlo Simulation Experiments

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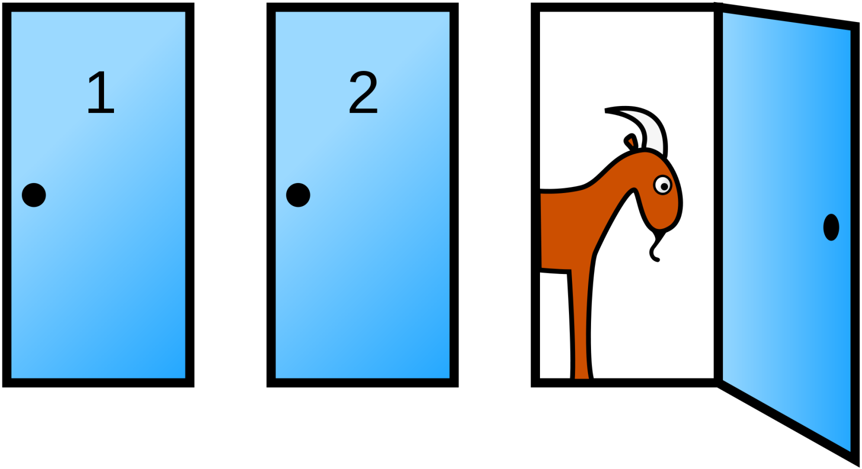
Monty Hall Problem

You're on a game show, and you're given the choice of three doors. Behind one door is a car; behind the others, goats.

You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?"

Is it to your advantage to switch your choice?


1
2

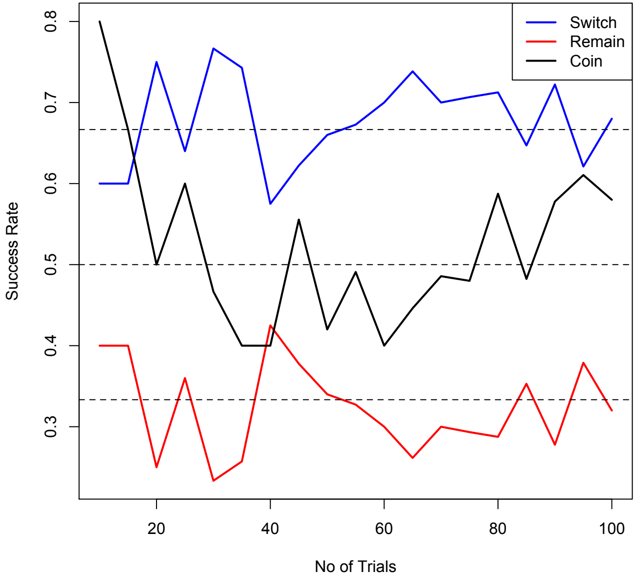
An experimental strategy

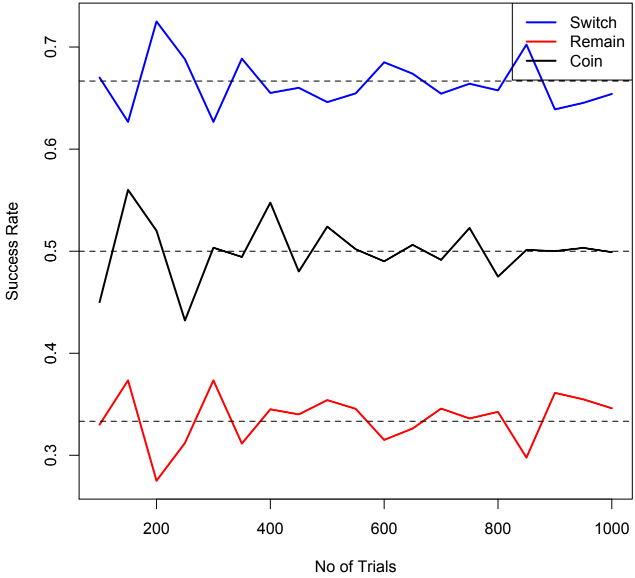
Just play the game!

A Statistician's strategy: Flip a coin to decide whether to switch or remain.

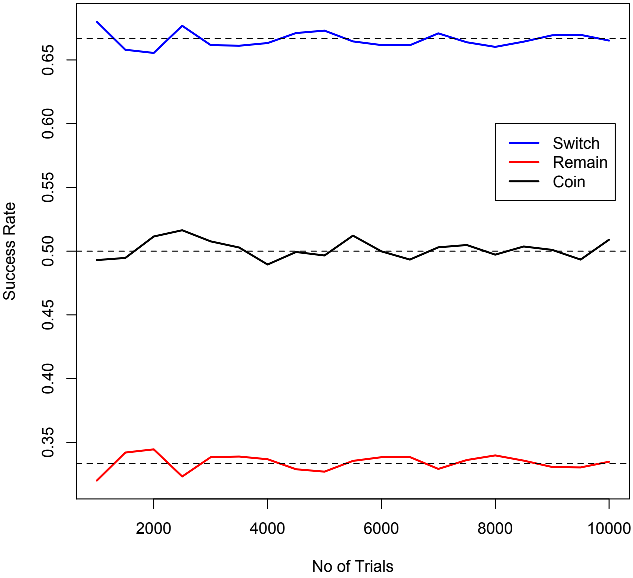
Repeat playing the game several times under each strategie (Switch, Remain, Coin).

Calculate success rate (proportion of wins).

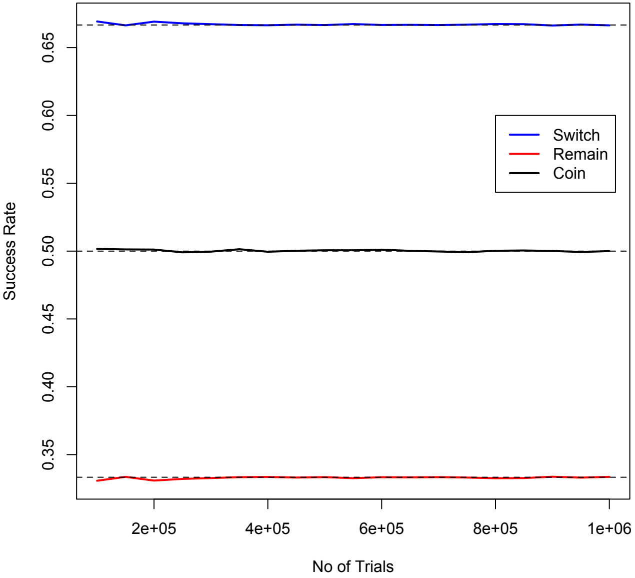

Switch
Remain
Coin
Success Rate
0.3
0.4
0.5
0.6
0.7
0.8
20
40
60
80
100
No of Trials

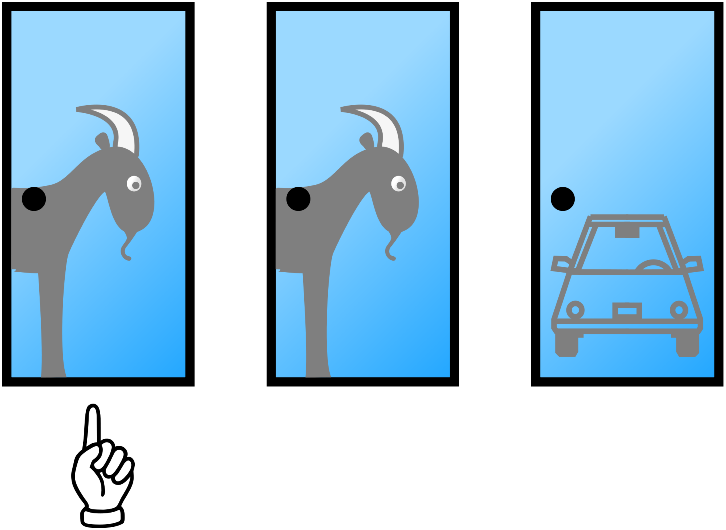


Switch
Remain
Coin
0.3
0.4
0.5
0.6
0.7
Success Rate
200
400
600
800
1000
No of Trials

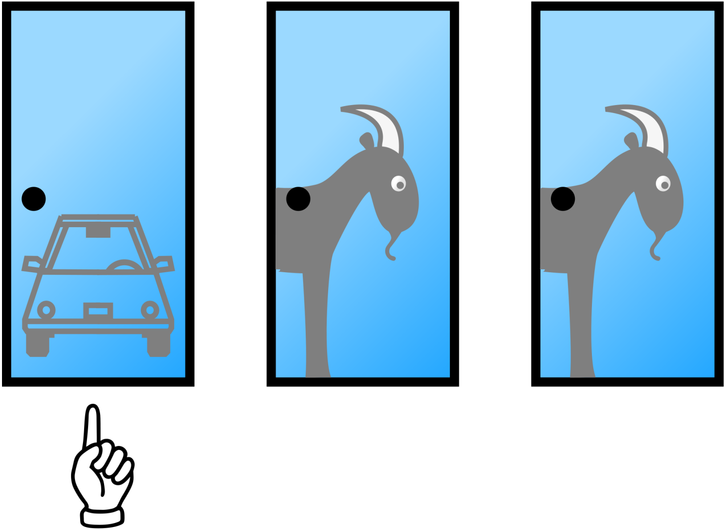


Switch
Remain
Coin
Success Rate
0.35
0.40
0.45
0.50
0.55
0.60
0.65
2000
4000
6000
8000
10000
No of Trials

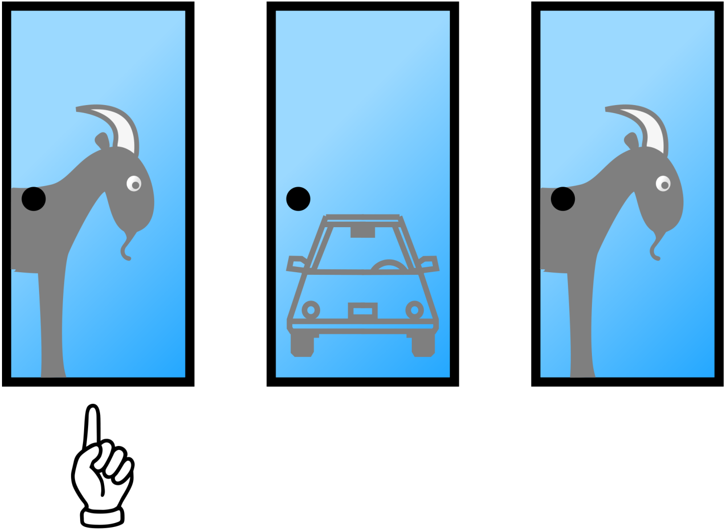

Switch
Remain
Coin
Success Rate
No of Trials
0.35
0.40
0.45
0.50
0.55
0.60
0.65
2e+05
4e+05
6e+05
8e+05
1e+06



Switch – Win; Remain – Lose



Switch – Lose; Remain – Win

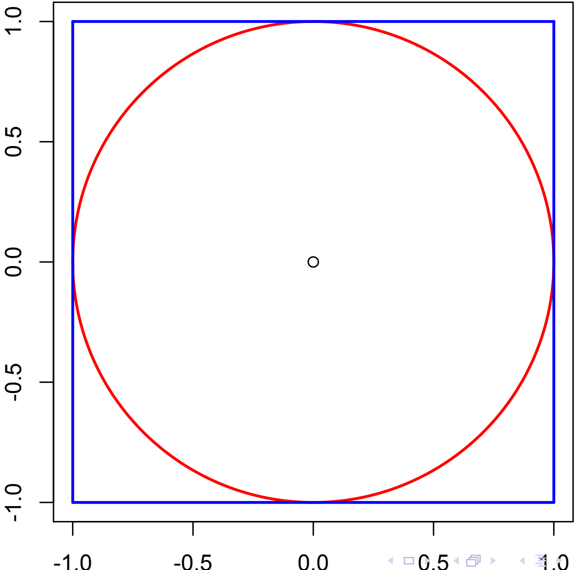


Switch – Win; Remain – Lose

Estimation of π

A Dumb Game of Darts

Game: Throw darts into the blue square.

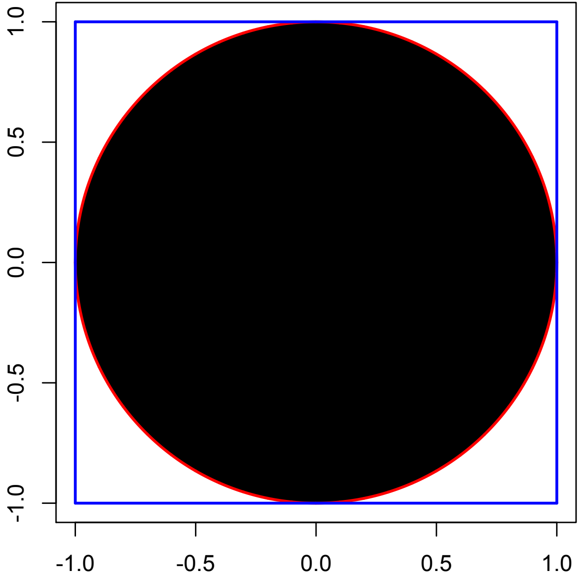


A Dumb Game of Darts

Good: We are sure that all the darts will land inside the square.

Bad: The darts land at *purely random* locations in the square.

Question: What is the proportion of darts that will land inside the red circle?



1.0
0.5
0.0
-0.5
-1.0
-1.0
-0.5
0.0
0.5
1.0

Theoretically

$$\text{Probability of landing in circle} = \frac{\text{Area of Circle}}{\text{Area of Square}} = \frac{\pi}{4}$$

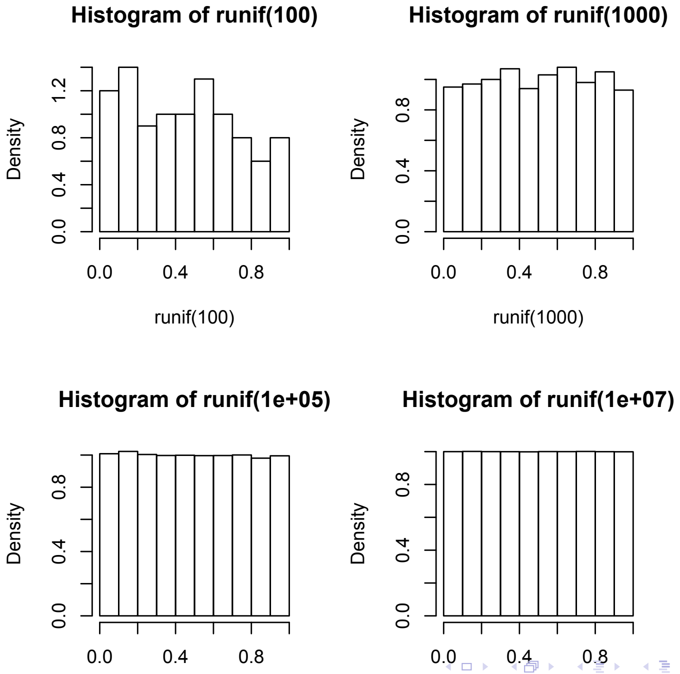
So, In the long run

$$\pi \approx 4 \times \text{the proportion of darts in the circle}$$

Question 1: How can we generate darts that land at random locations in the square?

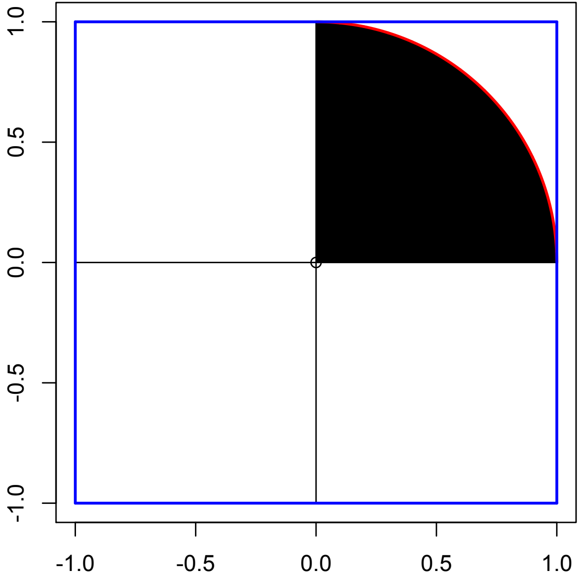
Question 2: How can we repeat the 'dart throwing' a large enough number of times to get a good approximation of π?


1
2
3
RANDOM
0
•
+/−


Histogram of runif(100)
Density
0.0
0.4
0.8
1.2
0.0
0.4
0.8
runif(100)
Histogram of runif(1000)
Density
0.0
0.4
0.8
0.0
0.4
0.8
runif(1000)
Histogram of runif(1e+05)
Density
0.0
0.4
0.8
0.0
0.4
0.8
Histogram of runif(1e+07)
Density
0.0
0.4
0.8
0.0
0.4
0.8

I actually used an R code ...

- Most, if not all, programming languages have a random number generator.
- These generate a bunch of numbers that are *uniformly* distributed over the interval $[0, 1]$.

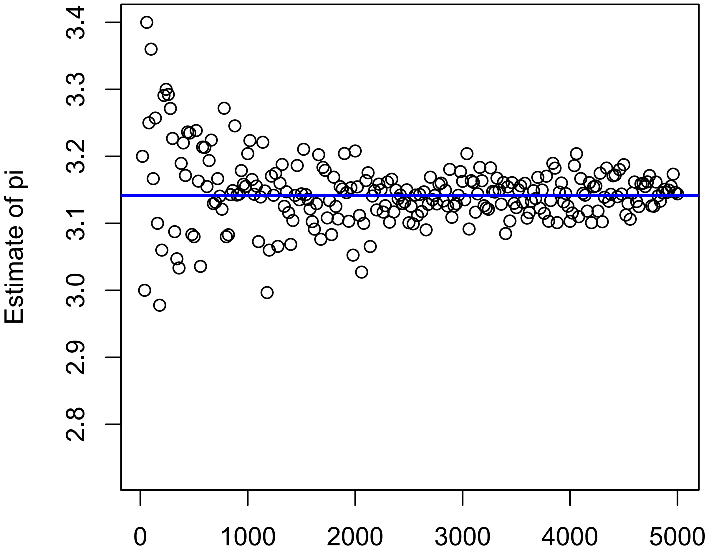


1.0
0.5
0.0
-0.5
-1.0
-1.0
-0.5
0.0
0.5
1.0

Algorithm for estimating π

1. Generate two uniform random numbers on $[0, 1]$, say, U_1 and U_2.
2. Let
$$X = \begin{cases} 1 & \text{if } U_1^2 + U_2^2 < 1 \\ 0 & \text{otherwise.} \end{cases}$$
3. Repeat 1 and 2 many times, say N times, and collect $X_1, X_2, \ldots, X_N$.
4. Now
$$\frac{\pi}{4} \approx \frac{X_1 + X_2 + \cdots + X_N}{N}$$

```
piest = function(N){
    U1 = runif(N)
    U2 = runif(N)
    count = rep(0,N)
    count[U1^2 + U2^2 < 1] = 1
    4*mean(count)
}
```


Estimate of pi
2.8
2.9
3.0
3.1
3.2
3.3
3.4
0
1000
2000
3000
4000
5000

Effect of Talking on Blood Pressure

- Does talking elevate blood pressure? This is a version of the 'white coat effect'.
- 16 patients with high BP were randomly assigned to one of two groups.
- Talking group : asked questions about their medical history and sources of stress in their lives before BP measurement
- Counting group : asked to count from 1 to 100 four times before BP measurement

Talking	104	110	107	112	108	103	108	118
Counting	110	96	103	98	100	109	97	105

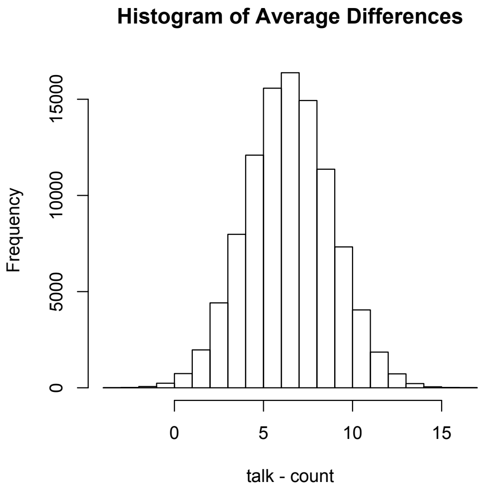
Now Bootstrap

This requires repetition.

Sample each sample 100,000 times and compute many (100,000) differences between averages.

Sort and pick an interval that contains 95% of the differences.

If 0 is not included in this interval, then claim with 95% certainty that there is a difference.



95% confidence interval: $(1.875, 11.25)$.

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